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The Higgs field and the resolution of the Cosmological Constant Paradox in the Weyl-geometrical Universe

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The nature of the scalar field responsible for the cosmological inflation is found to be rooted in the most fundamental concept of Weyl's differential geometry: the parallel displacement of vectors in curved space–time. Within this novel geometrical scenario, the standard electroweak theory of leptons based on the $SU(2)_L \otimes U(1)_Y$ as well as on the conformal groups of space–time Weyl's transformations is analysed within the framework of a general-relativistic, conformally covariant scalar-tensor theory that includes the electromagnetic and the Yang–Mills fields. A Higgs mechanism within a spontaneous symmetry breaking process is identified and this offers formal connections between some relevant properties of the elementary particles and the dark energy content of the Universe. An 'effective cosmological potential': V_{eff} is expressed in terms of the dark energy potential: $|V_\Lambda| \equiv M_\Lambda^2$ via the 'mass reduction parameter': $\zeta \equiv \sqrt{|V_{\mathrm{eff}}|/|V_\Lambda|}$, a general property of the Universe. The mass of the Higgs boson, which is considered a 'free parameter' by the standard electroweak theory, by our theory is found to be proportional to the mass $M_{\mathrm{eff}} \equiv \sqrt{|V_{\mathrm{eff}}|}$ which accounts for the measured cosmological constant, i.e. the measured content of vacuum-energy in the Universe. The non-integrable application of Weyl's geometry leads to a Proca equation accounting for the dynamics of a ϕ^ρ-particle, a vector-meson proposed as an an optimum candidate for dark matter. On the basis of previous cosmic microwave background results our theory leads, in the condition of cosmological 'critical density', to the assessment of the average energy content of the ϕ^ρ-excitation. The peculiar mathematical

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structure of V_{eff} offers a clue towards a very general resolution of a most intriguing puzzle of modern quantum field theory, the 'Cosmological Constant Paradox' (here referred to as the 'Λ-Paradox'). Indeed, our 'universal' theory offers a resolution of the Λ-Paradox for all exponential inflationary potentials: $V_\Lambda(T, \phi) \propto e^{-n\phi}$, and for all linear superpositions of these potentials, where n belongs to the mathematical set of the 'real numbers'. An explicit solution of the Λ-Paradox is reported for $n = 2$. The resolution of the Λ-Paradox cannot be achieved in the context of Riemann's differential geometry.

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1. Introduction: the Weyl geometry and the cosmological inflation

A huge step forward in theoretical cosmology, and a landmark of modern science, was the proposal by Starobinsky [1] followed by Guth [2], Linde [3], and Albrecht & Steinhard [4] of the *inflation*, an epoch of fast accelerating expansion of the early Universe that caused the Universe to expand through about 70 *e*-folds in a small fraction of a second. This expansion, driven by a scalar field called 'inflaton', was originally argued to solve the problem of why the Universe is so smooth at large scales. Over the years, the undeniable success of this idea was somewhat questioned by the failure of finding the physical mechanism underlying the nature of the *inflaton* concept [5–7]. In a recent letter, we claimed that the fundamental nature of this scalar field is indeed *geometrical*, based on the conformal differential geometry introduced by Hermann Weyl [8–10].[1] The Weyl geometry rests on a statement that may be cast in the simple form: 'all laws of physics are conformally covariant' (or *Weyl covariant* or, in short, *co-covariant*).

Our conjecture regarding the physics–geometry connection is as follows. According to Weyl, the parallel displacement from two infinitely nearby points P and $P + dP$ in space–time acts on the length ℓ of any vector by inducing a 'calibration' (i.e. 'gauge') change $\delta \ell = \ell \phi_\rho \, dx^\rho$, where ϕ_ρ is a universal 'Weyl vector', defined in the whole space–time. Indeed the structure of the Weyl geometry implies two forms: the *quadratic* Riemannian one, $g_{\rho\sigma} \, dx^\rho \, dx^\sigma$, $g_{\rho\sigma} = g_{\sigma\rho}$ being the metric tensor, and Weyl's *linear* one $\phi_\rho \, dx^\rho$, which is non-existent in Riemann's geometry. The parallel displacement is *integrable* iff a scalar 'Weyl potential' ϕ exists such as: $\phi_\rho = \partial_\rho \phi$. The *Weyl vector* $\phi_\rho(x)$ and the *Weyl potential* $\phi(x)$ defined in the four-dimensional (x^ρ) space–time play a basic role in this work as the inflaton *is identified* with $\phi(x)$. The inflationary physics is identified with geometry. The field ϕ is a gauge field since the Weyl geometry consists of an abelian, local, scale-invariance gauge theory implying the group of conformal transformations: $\{(\phi_\rho \rightarrow \phi_\rho + \partial_\rho \lambda(x)); (g_{\rho\sigma} \rightarrow e^{2\lambda} g_{\rho\sigma})\}$ [8,13]. Details on the Weyl geometry in the context of this work are given in [8,10,14,15] (see footnote 1).

All these concepts will be applied to a very general Weyl–Dirac conformal theory involving the mass m_{E} of an elementary particle, e.g. an electron [16]. The mass of the particle is expressed in the form of a 'mass field' $m_{\text{E}} \rightarrow k_{\text{E}} \cdot \mu(x)$, k_{E} being the dimensionless coupling constant, an intrinsic particle's property, and $\mu(x)$ a real scalar field, function of space–time, with weight $W(\mu) = -1$

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[1]Some useful mathematical results. $W(\tilde{L}) = +1$, $W(g_{\rho\sigma}) = +2$, $W(g^{\rho\sigma}) = -2$, $W(\sqrt{-g}) = +4$, $W(m \equiv \text{mass}) = -1$, $W(\bar{R}) = -2$. Here $g_{\rho\sigma}$ is the metric tensor, $g = \det(g_{\rho\sigma})$ and $\bar{R} = R + R_{\text{W}}$ is the sum of the Riemann and the Weyl curvature scalars. The Riemann covariant divergence of ϕ^ρ is $\phi^\rho_{|\rho} = \partial_\rho \phi + \Gamma^\rho_{\sigma\rho}\phi^\sigma$, where $\Gamma^\eta_{\sigma\rho}$ are the Christoffel symbols, while the *co-covariant derivative* of the scalar quantity X is: $D_\rho(X) = [\partial_\rho X - W(X)\phi_\rho X]$ so that $W(D_\rho X) = W(X)$. A useful result is: $g_{\rho\sigma|\sigma} = D_\sigma(g_{\rho\sigma}) = 0$. Our signature of $g_{\rho\sigma}$ is $(-, +, +, +)$. The Laplace–Beltrami operator acting on any function X is defined as: $\nabla_{\text{B}} X \equiv (1/\sqrt{-g})\partial_\rho(\sqrt{-g}\,g^{\rho\sigma}\partial_\sigma X)$. $\nabla_{\text{B}}\phi \equiv \phi^\rho_{|\rho}$. The Weyl curvature scalar in $D = 4$ is: $R_{\text{W}} = 6[\phi_\rho\phi^\rho - \phi^\rho_{|\rho}]$. The sign of the Ricci tensor: $R_{\rho\sigma}$ and of the curvature scalar: $\bar{R} = g^{\rho\sigma}R_{\rho\sigma}$ is determined by the choice of the lower index of the Riemann–Christoffel curvature tensor: R^i_{klm} to be saturated by the upper index. In modern general relativity texts (e.g. by P. A. M. Dirac, E. A. Lord, D. F. Lawden), the tensor: $R_{kl} = R^i_{kli}$ is adopted while: $R_{km} = -R_{kl}$ is adopted in other texts, e.g. by Weyl [8]. In this work, the first option is chosen leading to values of R and of R_{W} opposite to the ones often adopted in the context of the Weyl geometry [11,12]. Weyl [8] reports the first gauge theory, i.e. the prototype of all gauge theories today adopted in quantum field theory.

[17].[2] We define a dimensionless true constant: $(\alpha) = c^4(16\pi G\mu^2)^{-1}$ and assume that the ratio l_C/l_P between the particle Compton length $l_C = \hbar/mc$ and the Planck length is independent of position in space and time. Note that the quantity $c^4(16\pi G)^{-1}$ which appears in the Lagrangian of Einstein's theory cannot be regarded as a constant in the present conformal approach [9]. By applying Dirac's Lagrange-multiplier method, the most general scalar-tensor form of the co-covariant Lagrangian density in $D = 4$ can be expressed as [9,16]

$$\hat{L} = \sqrt{-g}\{\alpha\mu^2[\bar{R} + \eta(n)V_\Lambda(T,\phi)] - D_\rho\mu D^\rho\mu + |\lambda|\mu^4 - \tfrac{1}{4}\beta^2\phi_{\rho\sigma}\phi^{\rho\sigma} - \tfrac{1}{4}f^{\rho\sigma}f_{\rho\sigma} - \tfrac{1}{4}F^l_{\rho\sigma}F^{l\rho\sigma}\},$$
(1.1)

where $\bar{R} = (R + R_W)$ is the overall Riemann–Weyl curvature scalar, β a dimensionless coupling constant, D_ρ the Weyl co-covariant derivative (see footnote 1). The skew-symmetric tensors are: $\phi_{\rho\sigma} \equiv (\partial_\sigma\phi_\rho - \partial_\rho\phi_\sigma)$; $f_{\rho\sigma} \equiv (\partial_\sigma A_\rho - \partial_\rho A_\sigma)$ for the $U(1)_Y$ gauge field and $F^l_{\rho\sigma} \equiv (\partial_\sigma b^l_\rho - \partial_\rho b^l_\sigma + g\epsilon_{jkl}b^j_\rho b^k_\sigma)$ for the $SU(2)_L$ non-abelian gauge fields of the Yang–Mills theory. Here we introduce the gauge vector boson: A_ρ, i.e. the electromagnetic vector potential, for the group $U(1)_Y$ and the three components in the isospin space: $(b^1_\rho, b^2_\rho, b^3_\rho)$ of the gauge vector boson: b_ρ for $SU(2)_L$. Y is the 'weak hypercharge' operator. The term proportional to $|\lambda|$ is added for dynamical stability against unbounded field oscillations. The dimensionless quantity: $\eta(n) \equiv [(6 - \alpha^{-1}) \times n]$ is function of any real number: n. The potential $V_\Lambda(T, \phi)$ is related to the temperature (T)-dependent 'cosmological constant' Λ and accounts for the self-interaction of the scalar field ϕ [18]. The conditions implied by the Weyl symmetry on each co-covariant addendum X appearing in the expression of $\hat{L}$, i.e. $W(\hat{L}) = W(X) = 0$, impose well defined constraints on the function $V_\Lambda(T, \phi)$. For instance, since for any physical quantity: $X \to e^{\lambda(x)W(X)}X$, and because here: $W(V_\Lambda) = -2, W(\mu^2) = -2, \ W(\sqrt{-g}) = +4$, a possible expression to be inserted in equation (1.1) may be cast in the co-covariant form as an exponential: $V_\Lambda(T, \phi) \propto e^{-2\phi}$ or as a superposition of exponentials [8,10,19].[3]

In addition to the above analysis, we must impose the general condition that the action: $I_\phi = \int dx^4\sqrt{-g}L_\phi$ is stationary with respect to variations in ϕ, where the Lagrangian: $L_\phi \equiv [\tfrac{1}{2}g^{\rho\sigma}\partial_\rho\phi\partial_\sigma\phi - V_\Lambda(T,\phi)]$ expresses the effect of the inflation field in the $D = 4$ space–time [6]. This condition is expressed by

$$\nabla_B\phi = -V'_\Lambda(T, \phi),$$
(1.2)

∇_B being the Laplace–Beltrami differential operator [6] (see footnote 1) and $V'_\Lambda(T, \phi) \equiv \partial V_\Lambda(T, \phi)/\partial\phi$.

In the theory above, the Lagrangian $\hat{L}$, equation (1.1), is written in terms of the *geometrical* Weyl potential ϕ_W within a Weyl–Dirac scalar-tensor theory, e.g. according to the texts by Dirac [16, p. 410] and Lord [9, p. 197]. On the other hand, the Lagrangian L_ϕ is written in terms of the *physical* 'inflaton' field ϕ_I, e.g. according to the texts by Dodelson [5, p. 152] and Weinberg [6, p. 526]. According to the basic conjecture of our theory the two fields should be considered as complementary aspects of the same entity: $\phi_W \equiv \phi_I \equiv \phi$. Accordingly, there is no conflict between the corresponding Lagrangian theories but rather equation (1.2), i.e. the Euler–Lagrange result of the variation respect ϕ of L_ϕ, provides the necessary mathematical relation leading to the completion of the theory in closed form, i.e. with no approximations. Let us start with the *integrable* form of the Weyl theory, i.e. $\phi_{\rho\sigma} = 0$.

2. The Higgs field and the vacuum energy in the Universe

The wide conceptual scenario opened by the preceeding discussion and the structure of the Lagrangian $\hat{L}$, equation (1.1), offer the possibility of inquiring about the implications of the dark energy content of the Universe within some relevant aspects of the submicroscopic world of the elementary particles. The supposed mass generation properties of the Higgs field, and the

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[2]The 'mass field' is proportional to the corresponding 'Higgs field'. The book reports the following quote by Abdus Salam: 'The massless Yang–Mills particles 'eat' the Higgs particles (or field) in order to gain weight, and the swallowed Higgs particles become ghosts'.

[3]The exponential solution for the inflaton potential $V_\Lambda(\phi)$ proposed in [19, pp. 139 and 149] for the 'quintessence' field complies with our solutions that provide the conformal covariance of the theory and the resolution of the Λ-Paradox.

pervasiveness that parallels analogous aspects of the inflationary field, has already stimulated in recent years a wealth of research in the field referred to as 'Higgs inflation' [20–22].[4] In what follows we shall find that the connection between the two fields can be demonstrated in the context of the present conformally covariant theory in which the 'classical' general relativity approach based on the Lagrangian equation (1.1) is associated with a spontaneously broken $SU(2)_L \otimes SU(1)_L$ gauge theory. We shall briefly consider this theory in the framework of the standard electroweak 'theory of leptons' due to Weinberg [24] and Salam [25]. Let us introduce a *complex* iso-doublet of scalar fields:

$$\vec{\mu} \equiv \begin{pmatrix} \mu^+ \\ \mu^0 \end{pmatrix} \qquad (2.1)$$

that transforms as a $SU(2)_L$ doublet with heavy hypercharge: $Y_\mu = +1$ according to the Gell Mann–Nishijima relation for the electric charge: $Q = I_3 + \frac{1}{2}Y$. The weak-isospin projection I_3 and the weak-hypercharge are commuting operators: $[I_3, Y] = 0$ [26,27]. Next, we introduce in the expression of $\hat{L}$, equation (1.1), the following replacement: $D_\rho \mu D^\rho \mu \rightarrow (D_\rho \vec{\mu})^\dagger \cdot (D^\rho \vec{\mu})$, where D_ρ expresses the Weyl-covariant, i.e. co-covariant, derivative and the dot represents scalar vector multiplication in the isospin space. For the sake of simplicity, we still keep the symbols: $\mu^2 \equiv (\vec{\mu}^\dagger \cdot \vec{\mu})$, and $\mu^4 \equiv (\vec{\mu}^\dagger \cdot \vec{\mu})^2$ since these scalar quantities do not unsettle the *real* structure of the general relativity theory. The gauge co-covariant derivatives are

$$D_\rho \vec{\mu} = [\partial_\rho + \phi_\rho]\vec{\mu} + \frac{i}{2}[g'YA_\rho + g\hat{\tau} \cdot \hat{b}_\rho]\vec{\mu}, \qquad (2.2)$$

where the components of the vector $\hat{\tau}$: (τ_1, τ_2, τ_3) are the Pauli spin operators, g represents the coupling constant of the weak-isospin group $SU(2)_L$ and $g'/2$ represents the coupling constant of the weak-hypercharge group $U(1)_Y$. The presence of the inflaton field ϕ_ρ multiplied by $\vec{\mu}$ in the first *real* square bracket at the r.h.s. of equation (2.2) is due to the weight $W(\vec{\mu}) = -1$ within the co-covariant derivative (see footnote 1). Equation (2.2) reproduces the spontaneous symmetry breaking theory of leptons [26,27] by further introducing within the standard theory the formal change: $\partial_\rho \rightarrow [\partial_\rho + \phi_\rho]$. As shown by footnote[5], this change provides a contribution to the interaction between the Higgs and the inflaton fields we are now dealing with. As we shall see by inspection of the Lagrangian $\hat{L}$, equation (1.1), other far larger contributions to the same effect, proportional to α, are due to the $\phi_\rho \phi^\rho$ and $\phi^\rho_{|\rho} = -V'_\Lambda(T, \phi)$ terms in the curvature R_W (see footnote 1). In summary, and most interesting, all that shows that it is precisely the conformally covariant structure of Weyl's geometry that establishes the connection between the two universal fields, ϕ and μ, the protagonists of the present analysis.

Let us briefly outline the standard theory of leptons on the basis of the classic texts [26,27]. Consider in general terms the kinetic term of the dynamical equation for the scalar field $\vec{\mu}$:

$$\{(D_\rho \vec{\mu})^\dagger \cdot (D^\rho \vec{\mu}) - |\lambda|\mu^4\} = \{(\partial_\rho \vec{\mu})^\dagger \cdot (\partial^\rho \vec{\mu}) - \langle\mu\rangle^2\mu^2 - |\lambda|\mu^4\}. \qquad (2.3)$$

If the mass term $\langle\mu\rangle^2$ is negative, as suggested for instance by footnote 6 for $[\phi_\rho \phi^\rho > (-V'_\Lambda)]$, the continuous symmetry of the system's Hamiltonian does not coincide with the symmetry of the vacuum and the condition of dynamical 'spontaneous symmetry breaking' takes place. In the virtue of a theorem [28], a possible Goldstone boson is associated with a generator of the gauge group that does not leave the vacuum invariant. We investigate this case by choosing the following vacuum expectation value of the scalar field equation (2.1):

$$\langle\phi\rangle_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}, \qquad (2.4)$$

[4]Note also [23]: refer to the large bibliography in this text.

[5]In the context of $\hat{L}$, we can set the value of the covariant divergence: $(1/\sqrt{-g})\partial_\rho[\sqrt{-g}\phi^\rho(\vec{\mu}^\dagger \cdot \vec{\mu})] = 0$. Then: $\phi^\rho_{|\rho}(\vec{\mu}^\dagger \cdot \vec{\mu}) = -\phi^\rho \partial_\rho(\vec{\mu}^\dagger \cdot \vec{\mu})$ and: $[(\partial_\rho + \phi_\rho)\vec{\mu}^\dagger] \cdot [(\partial^\rho + \phi^\rho)\vec{\mu}] = \{(\partial_\rho \vec{\mu})^\dagger \cdot (\partial^\rho \vec{\mu}) + [\phi_\rho \phi^\rho + V'_\Lambda(\phi, T)]\mu^2\}$, in virtue of equation (1.2). As we see by inspection of the Lagrangian $\hat{L}$, equation (1.1), the other important contribution to the symmetry breaking is due to the term: $(\alpha\mu^2 \cdot \vec{R})$.

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where the vacuum field is $v = \sqrt{-\langle\mu\rangle^2/|\lambda|}$. The vacuum is left invariant by any group generator G if: $G\langle\phi\rangle_0 = 0$. In our case, we find all the $SU(2)_L$ and $U(1)_Y$ group generators: $\hat{\tau}_i$ $(i = 1, 2, 3)$ and Y operating on $\langle\phi\rangle_0$ break the symmetry of the vacuum. However, the $U(1)_{EM}$ symmetry generated by the electric charge preserves the invariance since: $Q\langle\phi\rangle_0 \equiv \frac{1}{2}(\tau_3 + Y)\langle\phi\rangle_0 = \binom{0}{0}$. The photon, therefore, remains massless and the other three gauge bosons will acquire mass. These are the heavy bosons: $W_\rho^\pm \equiv [b_\rho^1 \mp ib_\rho^2]/\sqrt{2}$ and $Z_\rho \equiv [-g'A_\rho + gb_\rho^3]/\sqrt{g^2 + g'^2}$. Upon expansion of the Lagrangian (2.3) about the shifted minimum of the Higgs potential, we can investigate the small oscillations around the vacuum v of of the 'Higgs field', this one expressed by the field θ. This dynamics is expressed by the Lagrangian for small oscillations:

$$\bar{L} = \frac{1}{2}[(\partial_\rho\theta)(\partial^\rho\theta) + 2\langle\mu\rangle^2\theta^2] + \frac{v^2}{8}[g^2|b_\rho{}^1 - ib_\rho^2|^2 + (-g'A_\rho + gb_\rho^3)^2] + \cdots \qquad (2.5)$$

plus interaction terms. As shown by this equation, the Higgs field has acquired a $(\text{mass})^2 \equiv (M_H)^2 = 2|\langle\mu\rangle|^2$. The above sentences express in a very summary form the well-known results of the standard electroweak theory.

We may now inquire about the effects on the theory of the change: $\partial_\rho \rightarrow [\partial_\rho + \phi_\rho]$ introduced in our present analysis by the co-covariant derivative (2.2) as well of the general relativistic structure of the overall co-covariant Lagrangian $\hat{L}$ (1.1). Note first that nothing in the standard formulation of the electroweak theory based on the $SU(2)_L \otimes U(1)_Y$ group specifies the mass of the Higgs boson and none of the conceivable applications to the conventional processes depends in any way upon the value of M_H. It may therefore appear that M_H can indeed be considered a 'free parameter' of the standard electroweak theory even if general constraints have been considered by some authors by imposing certain requirements of internal consistency [13].[6]

This is not the case with our present theory. As equation (2.3) is formally included into the extended Lagrangian equation (1.1), we can easily transfer to the last one all the physical conceptions and the theoretical considerations addressed so far to equation (2.3). In particular, the Riemann curvature R and the Weyl curvature R_W appearing in equation (1.1) are now involved in the spontaneous symmetry breaking scenario of the electroweak theory. On the other hand, they can also be expressed in terms of the relevant cosmological quantities, i.e. the inflation potential: $V_\Lambda(T, \phi)$, its ϕ-derivative: $V'_\Lambda(T, \phi) = -\nabla_B\phi$ and the inflaton vector field: ϕ_ρ. Consequently, and very important, the dynamical behaviour of all interactions affecting the properties of the elementary particles, including the Higgs field, is directly determined by these cosmological quantities, and then by the overall dynamics of the Universe. We believe that this is an interesting result.

In view of what we believe to be the most interesting result of the present work, i.e. the resolution of the 'Cosmological Constant Paradox' (referred to as 'Λ-Paradox', hereafter), in what follows we shall mostly deal with the exponential potential functions of ϕ: i.e. $V_\Lambda(T, \phi) \propto e^{-n\phi}$ and to the linear superpositions of these functions. There n is a 'real number', i.e. either positive or negative, integer or fractional [19].[7] Within this restriction, we claim the 'universality' of the theory and then in what follows we shall consider only the case: $n = 2$, as an example. For any other (n), the theoretical steps can be reproduced identically albeit the final numerical results of the calculations will be different.

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[6]The Cosmological Constant Paradox was analysed within the standard electroweak theoretical context by Quigg [13]. In his work, our equation (2.7) is presented in a different form: $M_H^2 = (8 \cdot v^{-2} \cdot \rho_H)$, where $v = \sqrt{|\mu|^2/|\lambda|} = (\sqrt{2}G_F)^{-1/2} \sim 246$ (GeV) is the vacuum field appearing in the isodoublet (2.1) and: $G_F = 1.66 \times 10^{-5}(\text{GeV}^{-2})$ is the Fermi constant. From the perspective of general relativity, this amounts to adding to Einstein's equation a cosmological constant: $\Lambda = [(8\pi G/c^4) \cdot \rho_H]$. By inserting the measured value of $M_H = 129.09$ (GeV/c^2), the theoretical value given by Quigg of the vacuum-energy density is: $\rho_H \sim 1.2 \times 10^8$ (GeV/c^2)^4. The corresponding value evaluated by our theory is: $\rho_H \sim 4.8 \times 10^8$ (GeV/c^2)^4. We consider the fair agreement existing between the two results a good test for our present theory.

[7]The Riemann curvature R, expressed by equation (3.2), is obtained by variation respect $(g_{\rho\sigma}, \phi_\rho, \mu)$ of the Lagrangian $\hat{L}$. Then, both sides of the Einstein equation are multiplied by the metric tensor, by keeping in mind that, in $D = 4$: $g_{\rho\sigma}g^{\rho\sigma} = 4$. In the general case is found: $V_{\text{eff}} = [V_\Lambda + (n)^{-1}V'_\Lambda]$, the cosmological constant: $\Lambda = [\eta(n)/2 + \bar{\eta}/2]$. $\eta(n) \equiv [(6 - \alpha^{-1}) \times n]$, $\bar{\eta} \equiv [3 - (2\alpha)^{-1}]$ and $n =$ any real number, positive or negative, integer or fractional. For $n \neq 2$, the co-variance of the solution of the Λ-Paradox is easily attained by the multiplication of $\tilde{C}(T)$ with $(M_P)^\gamma$ with a suitable γ-exponent.

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Let us first define an 'effective cosmological potential':

$$V_{\text{eff}}(T,\phi) \equiv \left[V_\Lambda(T,\phi) + \frac{1}{2} \frac{\partial V_\Lambda(T,\phi)}{\partial \phi} \right], \tag{2.6}$$

that can be either positive or negative depending on the sign and size of the proportionality parameters of $V_\Lambda(T,\phi)$ and/or of its ϕ-derivative. In the case of a negative derivative, a very small value of $|V_{\text{eff}}|$ may result from the sum of two very large contributions with opposite sign. As said, the contribution V_Λ is the vacuum energy, i.e. the dark energy content of the Universe, conceptually connected with Einstein's 'cosmological constant' [18]. The quantity $|V_{\text{eff}}| \equiv M_{\text{eff}}^2$ represents the corresponding *measured* quantity. The above effect, expressed by the size of the 'mass-reduction parameter': $\zeta \equiv \sqrt{|V_{\text{eff}}|/|V_\Lambda|} \ll 1$, indeed a general property of the Universe, can lead to a consequence of cosmological relevance since it represents a clue towards the resolution of the celebrated 'Λ-Paradox' [29–31].[8,9,10] A conformally covariant solution of the paradox based on equation (2.6) will be given later in this paper.

In agreement with our programme and with the discussion above, we turn our attention to the evaluation of the mass of the Higgs boson, M_H. By considering equation (2.5), by collecting all terms proportional to μ^2 in equation (1.1) and by expressing the Riemann curvature scalar in terms of $V_\Lambda(T,\phi)$, $V'_\Lambda(T,\phi)$ and $\phi_\rho \phi^\rho$ (see footnote 8), we find the expression

$$M_H^2 = 4|(1-6a)V_{\text{eff}}|. \tag{2.7}$$

We find that the spontaneous symmetry breaking condition implies: $V_{\text{eff}} < 0$.

In conclusion, the present theory, which lies on Weyl's geometrical foundations, prescribes that the value of the mass of any elementary particle belonging to the submicroscopic world depends on the average vacuum energy content of the Universe. This because it is precisely the Higgs field that is the source of the mass of all elementary quantum particles [35–37].

The conceptual relevance of the 'effective cosmological potential' may be further enlightened by carrying out the variation respect $(g_{\rho\sigma}, \phi_\rho, \mu)$ of the Lagrangian $\hat{L}$ expressed by equation (1.1). For the sake of completeness, we also include here the energy-momentum tensor $T_{\rho\sigma}$ due to external, unspecified matter and fields. The Euler–Lagrange equation consists of Einstein's equation:

$$R_{\rho\sigma} - \tfrac{1}{2} g_{\rho\sigma} R - g_{\rho\sigma} \bar{\eta}[2V_{\text{eff}}(T,\phi) + \tfrac{1}{2}\partial_\rho\phi\partial^\rho\phi] = K^2 T_{\rho\sigma}, \tag{2.8}$$

where $\bar{\eta} \equiv [3 - (2\alpha)^{-1}]$. This equation is interesting because it shows that the generally very large 'inflation potential' $|V_\Lambda(T,\phi)|$ appearing in the Lagrangian $\hat{L}$, equation (1.1), is replaced here by the far smaller: $|V_{\text{eff}}(T,\phi)|$. The expression within the square brackets in (2.8) is the *measured* 'cosmological constant', Λ:

$$\Lambda \equiv \bar{\eta}[2V_{\text{eff}}(T,\phi) + \tfrac{1}{2}\partial_\rho\phi\partial^\rho\phi]. \tag{2.9}$$

As we shall see $|V_{\text{eff}}(T,\phi)| \ll |V_\Lambda(T,\phi)|$, and then $\zeta \ll 1$. We stress here that the formal replacement $|V_\Lambda(T,\phi| \to |V_{\text{eff}}(T,\phi)|$ is precisely due to the application of the dynamical equation (1.2) to the expression of the Weyl curvature R_W given in footnote 1. If the Weyl curvature is considered non-existent in the source lagrangian $\hat{L}$, equation (1.1), as assumed in the context of Riemann's differential geometry (where: $R_W = 0$), the replacement $|V_\Lambda(T,\phi)| \to |V_{\text{eff}}(T,\phi)|$ is not realized since $\Lambda = 0$. We believe that this is precisely the origin and the real essence of the Λ-Paradox. Indeed, we shall see that the above replacement is the key argument for the resolution of the paradox we shall propose later in this paper.

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[8]The following sentence is reported in [29, p. 753]: 'A distinguished colleague said to me recently: The cosmological constant paradox is more than a paradox: it's a profound public humiliation of theoretical physicists'.

[9]Various aspects of the Λ-Paradox have been analysed also by [32–34].

[10]It has been suggested by an anonymous referee that the requirement $\zeta \ll 1$ for the ratio of the potentials can be related to the 'slow roll' requirement of inflation (D. Baumann, TASI Lecture).

3. A meson vector field: dark matter?

If the 'integrability condition' of the Weyl geometry, i.e. $\phi_\rho = \partial_\rho \phi$ and $\phi_{\rho\sigma} = 0$, is relaxed, the quantities ϕ and ϕ_ρ must be considered independent variables of the theory and this one gets even richer.[11] In particular, the inner structure of the Weyl geometry does not change, since the parallel displacement of vectors involves directly the vector field ϕ^ρ, as said. Leaving aside all complications and deferring an exact analysis to future work, we may immediately apply the mathematical methods adopted in the previous sections to the Lagrangian $\hat{L}$, equation (1.1). The variation respect to the 'Weyl vector' $\phi_\rho(x)$ leads, via the gauge-fixing $\partial_\rho \phi^\rho = 0$, to the following Proca equation expressing the dynamics of a massive vector-meson, ϕ^ρ:

$$\left[\nabla_B + \left(\xi \frac{M_P c}{\hbar} \right)^2 \right] \phi^\rho = 0, \tag{3.1}$$

where $M_P = 1.22 \times 10^{19} (\text{GeV}/c^2)$ is the Planck mass and $\xi = \sqrt{2}(\beta)^{-1} \ll 1$ expresses the coupling of the particle to gravitons. It has been found that the massive meson ϕ^ρ, while strongly interacting with gravitons, does not interact with any spin-$\frac{1}{2}$ or spin-1 elementary particle of the standard model. In other words, it is not coupled minimally to photons, hadrons and light or massive leptons [38,39]. All these properties make this geometrical entity eligible for being considered an optimum candidate for cold dark matter (CDM), the elusive object which is now under investigation in a large number of laboratories around the world. As is well known, the CDM may consist of a weakly interacting massive particle (WIMP), a stable SUSY particle, a light neutralino with a mass of the order of 10^2 (GeV/c^2) or even an 'axion' particle with a mass as low as 10^{-5} (GeV/c^2) [19,40,41].[12] Large concentrations of CDM have been detected in zones of the Universe characterized by a large inhomogeneous gravitational energy density. Indeed the CDM predominately clusters on the scale of galaxies. In summary, today the size of the mass and any other physical property of CDM is inferred from some sophisticated quantum theory based on an attributed 'physical model'.

Waiting for any novel, convincing experimental evidence, we refrain from considering further the putative identification of CDM with the ϕ^ρ-particle. We tend nevertheless to believe, according to a perspective biased by our 'geometrical model', that the mass of this particle, i.e. the value of the parameter ξ in equation (3.1), must depend, once again, on a property of the Universe via some dynamical mechanism similar to the one already considered in this work. If the exact size of the mass of the ϕ^ρ-particle is presently out of reach, we are nevertheless able to determine the average size of this quantum excitation in the Universe. By a standard variational procedure, we evaluate by our theory the average curvature of the Universe (see footnote 8):

$$R = 2\bar{\eta}[4|V_{\text{eff}}| - \phi_\rho \phi^\rho]. \tag{3.2}$$

This equation is interesting since it shows the relative sizes of the two dominant contributions to the Universe curvature, i.e. the vacuum-energy, V_{eff} and the energy V_{DM} associated with the vector-meson ϕ^ρ. The average density of the ϕ^ρ field is

$$M_{\text{DM}} \simeq \sqrt{\langle \phi_\rho \phi^\rho \rangle} \simeq 2\sqrt{|V_{\text{eff}}|}, \tag{3.3}$$

since the energy density of the Universe is 'critical' and the Universe is approximately 'flat', i.e. $R \simeq 0$. This was indeed the main result of the BOOMERANG experiment carried out on cosmic microwave background by Paolo De Bernardis and his group in the year 2000 [42]. These results were confirmed by recent measurement carried out by the PLANCK mission [43].

In conclusion, we propose here the following conjecture of cosmological relevance: 'The result of all experiments carried out on the cosmic vacuum-energy, i.e. dark-energy or zero-point-energy or cosmological-constant, by our terrestrial or extraterrestrial measurement apparata (i.e. carried by orbiting satellites or spatial vehicles) always consists of the reduced value of the

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[11]In particular, the cosmological potential should now be dependent both on ϕ and ϕ^ρ, i.e. $V_A(T, \phi, \phi^\rho)$.

[12]Refer, for CDM references, to the very large bibliography given in these texts.

effective cosmological potential V_{eff} and never of the full value of V_Λ. This last quantity, a quantum-mechanical property of the Universe, is inaccessible to the human perception'. The conjecture is proved by the results of the BOOMERANG, PLANCK and other recent astrophysical experiments. It is also consistent with our equations: (2.7), (3.2) and (3.3).

As an alternative solution, the above conjecture may raise a profound epistemological and gnoseological issue whether the very *calibration* process within the parallel displacement of vectors in curved spaces is found to determine at a very fundamental level all measurement processes, i.e. is *an intrinsic feature* of all measurement processes, and reveals itself when the experiments are carried out over cosmological spaces and times.

4. The Cosmological Constant Paradox and its solution

The following quote by Richard Feynman is enlightening [44, p. 246]: 'Such a mass density would, at first sight at least, be expected to produce very large gravitational effects which are not observed. It is possible that we are calculating in a naive manner, and, if all of the consequences of the general theory of relativity (such as the gravitational effects produced by the large stresses implied here) were included, the effects might cancel out; but nobody has worked all this out. It is possible that some cutoff procedure that not only yields a finite energy density for the vacuum-state but also provides relativistic invariance may be found. The implications of such a result are at present completely unknown'.

Let us analyse the Λ-Paradox in 'a naive manner', as in Feynman's words. The vacuum energy in the Universe E_{vac}, i.e. the 'zero-point energy' of the quantum fields associated with all existing quantum particles is evaluated by quantum field theory (QFT). For simplicity, we consider here only the 'photon', which is the subject of quantum electro-dynamics (QED), the chapter of QFT accounting for the electromagnetic phenomena. To carry out the calculation of E_{vac}, we should first evaluate the spatial density ρ_{em} of the available $\vec{k}$-modes, i.e. of the spatial vectors over which the photons propagate in the free space. Afterwards, each mode, which is modelled by QED as a quantum-mechanical oscillator, is multiplied by 2-times (because of the two orthogonal polarizations) the oscillator's 'zero-point energy', which is $h\omega/2$, where h is the Planck constant. The frequency ω is taken to range from zero to a cutoff that may be assumed to be determined by the Planck length, $l_{\text{P}} = 1.616 \times 10^{-33}$ (cm): $\omega_c = (2\pi c/l_{\text{P}})$. At last, the resulting spatial energy density must be multiplied by the volume of the Universe W_{U} in order to get the final result: E_{vac}. We can amuse ourselves by carrying out the sequence of these operations, and more than that, by putting numbers at the end of every step of the calculation. I limit myself to give here the final result. The size of final vacuum-energy content is found: $E_{\text{vac}} = 2\pi\hbar c/L_{\text{P}}^4 \times W_U = 2\pi c^7/\hbar G^2 \times W_{\text{U}} = 2.9 \times 10^{98}$ (J cm^{-3}) $\times W_{\text{U}}$ (cm^3). Since the linear size of our Universe evaluated by a cosmic microwave background analysis is: $L_{\text{U}} \sim 124$ billions of light-years, the volume is: $W_{\text{U}} \sim (1.61 \times 10^{87})$ (cm^3). Then, the value of the content of the vacuum-energy, due only to photons, is: $E_{\text{vac}} \sim 5 \times 10^{185}$ (J) $\sim 3 \times 10^{204}$ (eV). To be more conservative, the frequency cutoff ω_c could be determined by the size of the Compton wavelength of the proton: $\lambda_c \simeq 2 \times 10^{-14}$ cm. In this case, all the numerical figures for the energies and for the energy densities given above should be multiplied by the factor $\sim 10^{-76}$. Even in this case, the size of all numbers keeps being impressive! We should remember here that in the vast realm of modern science, the QED theory and, in particular, the 'vacuum-field' concept were the most tested paradigms, ever. Uncountable and impressively precise experiments involving the atomic physics, the optical spontaneous emission, the Casimir effect, the atomic spectra, the Lamb's shift, the electron's magnetic moment, etc., were the landmarks of the great success of the twentieth century physics. The calculation carried out for photons should now be extended to the other existing particles and the corresponding energy contributions should be added, without any reasonable chance of mutual cancellations.

Interestingly enough, the Λ-Paradox was also analysed in the framework of the electroweak theory of leptons by Quigg [13].

The Λ-Paradox consists of the mysterious, humongous discrepancy existing between the enormous size of the overall *calculated*: $(E_{vac}/c^2)^2 = |V_\Lambda(T,\phi)|$ and the very small size of the 'cosmological constant' Λ, equation (2.9), *measured* today.

Our present theory offers a conformally covariant solution of the Λ-Paradox. Consider the following expression for the inflationary potential which accounts for the overall vacuum-energy content of the Universe calculated by the standard methods of QFT as shown above:

$$V_\Lambda(T,\phi) = \tilde{C}(T) \times \exp\left[2(\epsilon - 1)\right]\phi, \tag{4.1}$$

where $\tilde{C}(T)$ is assumed independent of $\mu(x)$ and of $g_{\rho\sigma}(x)$. We have already noted that, since for any physical quantity: $X \to e^{\lambda(x)W(X)}X$, and $W(\sqrt{-g}) = +4$, the above solution corresponding to $n = 2$ is indeed a conformally covariant solution (for $\epsilon = 0$) (see footnote 8). The quantity ϵ is a dimensionless real number: $\epsilon \approx 0$. Plugging equation (4.1) into equation (2.6) gives the ratio: $|V_{eff}|/|V_\Lambda| = \epsilon$. This is indeed a most drastic 'mass-reduction' effect with the parameter: $\zeta = \sqrt{|\epsilon|} \sim 10^{(-210)}$, where this number only accounts for the electromagnetic field considered by the above numerical evaluation of E_{vac} [31]. By this argument, the enormous content of vacuum-energy in the Universe, calculated by the QFT methods as shown above, and expressed by $|V_\Lambda(T,\phi)|$ is made consistent by our theory, i.e. by our equations (2.6) and (4.1), with the very small value of $|V_{eff}(T,\phi)|$ and then, of the 'effective' cosmological constant, Λ, equation (2.9), *measured* today [5–7]. Therefore, the Λ-Paradox is resolved by our theory for $n = 2$.

As already stressed, for any real number (n) the resolution of the Λ-Paradox is achieved quite generally by our theory for any exponential potential function: $V_\Lambda(T,\phi) \propto e^{-n\phi}$ by inserting in the Lagrangian expressed by equation (1.1) the corresponding value of the parameter $\eta(n)$. In the case of a linear superposition: $\sum_j V_\Lambda^j(T,\phi)$ of exponential functions: $V_\Lambda^j(T,\phi) \propto e^{-n_j\phi}$, where n_j are generally uncorrelated real numbers, it suffices to plug within the square brackets of the Lagrangian $\hat{L}$, equation (1.1), the sum: $\sum_j \eta(n_j)V_\Lambda^j(T,\phi)$.

The Λ-Paradox cannot be resolved in the context of Riemann's differential geometry.

5. Conclusion

In conclusion, by our present work we have established several bridges between at least three cultural domains of the scientific endeavour that traditionally are rather disconnected: the foundations of the differential geometry, here focused on the inspiring ideas by Hermann Weyl, the general relativistic cosmology, and the modern field theory of the elementary quantum particles. Our aim was to search, by a unitarian perspective, for any possible conceptual and theoretical connection existing between the pervasive quantum fields that actively dwell in the most remote corners of the Universe.

As a final comment to our somewhat unusual proposal let us identify here some key arguments of the underlying logical scenario. (i) The main result of our work is the expression of V_{eff}, equation (2.6), by the inclusion of the ϕ-derivative: V'. This is possible in virtue of equation (1.2), indeed the key equation of our work. (ii) Since equation (1.2) is the Euler–Lagrange equation of a variational procedure on the 'physical' Lagrangian L_ϕ, the quantity ϕ must be a 'physical' field, and not a mere 'geometrical' entity. Otherwise, equation (1.2) would be meaningless. (iii) The same 'physical' equation (1.2) establishes a direct and necessary link from physics to geometry via the expression $\phi^\rho_{|\rho}$ appearing in the Weyl curvature R_W. All this implies a lucky intersection of several nicely interweaved physical-geometrical concepts. Riemann's geometry, indeed a beautiful theory, proves to be *too simple* within our scenario. The lack in its foundational premises of the 'calibration', i.e. of the 'gauge' concept, moves it out of the game.

In summary, the geometrical mechanism proposed by this work represents a unifying scenario by which a unique quantum field appears to play, by different routes and under different forms, an essential role in determining the evolution of the Universe 'at large' as well as at the

microscopic level and via the dynamics of the Weyl scalar curvature R_W, of the everyday quantum phenomenology [10,45–53].[13] This appears to be a glimpse into quantum gravity.

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