Core-collapse supernovae (CCSNe) are the explosions that attend the deaths of massive stars. Despite decades of research, several aspects of the mechanism that drives these explosions remain uncertain and the subjects of continued investigation. In this short review, I will give an overview of the CCSN mechanism and current research in the field. In particular, I will focus on recent results from three-dimensional simulations and the impact of turbulence and detailed non-spherical progenitor structure on CCSNe. This contribution is based on a talk given at the ‘Bridging the Gap’ workshop at Chicheley Hall on 2 June 2016.

This article is part of the themed issue ‘Bridging the gap: from massive stars to supernovae’.

1. Introduction

Stars more massive than about $8M_\odot$ end their lives as incredibly luminous explosions known as core-collapse supernovae (CCSNe). Observationally, this is a fact. We have directly observed massive stars erupt into CCSNe and then disappear after the explosion has faded. The most famous case of direct observation is SN 1987A [1], but we have now directly identified a few dozen CCSN progenitors [2,3]. Theoretically, explaining the mechanism that drives these massive stellar explosions, however, has been a challenge. In this brief review, I will give an overview of the investigation into the CCSN mechanism with a particular focus on recent results in three dimensions (3D). This is a very broad topic and so the present review will not be exhaustive. The particularly inquisitive reader should also see more complete reviews [4–10], or refer to the Handbook of Supernovae (http://www.springer.com/us/book/9783319218458).

CCSNe are a singularly important phenomenon in the Universe for several reasons. First and foremost,
they are principal drivers of cosmic chemical evolution. During the Big Bang, only H and He, with a smattering of Li, were synthesized [11]. With the exception of the elements around the iron peak, which are produced by CCSNe’s brighter but rarer cousins, Type Ia SNe, a large fraction of heavy elements are ultimately traced back to CCSNe. This includes the elements crucial to life. By mass, humans (and most other forms of known life) are about 60% O, most of which was made in the cores of massive stars and spread throughout the Universe by their resulting explosions. CCSNe may also be the site of the rapid neutron capture process (r-process) that produced many of the extremely heavy elements above atomic mass of approximately 70, although this is not settled and current research favours neutron star mergers for the production of at least the second and third peaks of the r-process [12–15]. CCSNe are also intrinsically, if mysteriously, linked to long gamma-ray bursts [16]. A complete understanding of stellar core collapse is crucial for understanding the scenarios that give rise to neutron stars, pulsars, magnetars and stellar-mass black holes including the particularly massive black holes that have recently been detected by Advanced LIGO [17–19].

Despite the incredible importance of CCSNe to our understanding of multiple aspects of the Universe, we still do not fully understand the processes that drive these explosions. The CCSN problem has been a perennial challenge in theoretical astrophysics for decades. The persistence of the struggle, however, is not an indication of a lack of progress. Our understanding of the physics of the CCSN mechanism has continually and tremendously evolved and, in particular, recent years have seen enormous advancement in the goal of developing a predictive theory for massive stellar death.

One of the largest impediments to solving the CCSN problem is the wide range and complexity of the physics involved. The CCSN mechanism involves 3D magnetohydrodynamics (MHD), general relativistic gravity, the transport of neutrinos in a dynamic environment, and complicated and uncertain microphysics such as the equation of state (EOS) of nuclear matter and the interaction rates of neutrinos with matter under extreme conditions. Any one of these aspects is a challenge in its own right but taken together they represent a grand challenge in astrophysics. Indeed, only in the last few years have high-fidelity 3D simulations of this process been feasible. And it is the advent of 3D simulation that is currently driving a renaissance in the field characterized by accelerated progress and success.

In this brief review, I will focus primarily on progress made in the investigation of the neutrino mechanism for CCSNe. Many other mechanisms have been proposed but the neutrino mechanism is widely considered the most promising and is the most well studied. This review will proceed as follows. In §2, I will give a whirlwind tour of the history of the investigation into the CCSN mechanism. The essential physics that leads to massive stellar core collapse is presented in §3. Section 4 discusses one-dimensional (1D) simulations of CCSNe and how artificially driven explosions can be compared to observations. Section 5 reviews progress in two dimensions (2D) and the emerging preponderance of successful self-consistent neutrino-driven explosions. I review the recent developments in simulating CCSNe in 3D in §6 and discuss the importance of turbulence in setting the dynamics of CCSNe in §7. In §8, I discuss the dramatic impact that nonspherical structure in the progenitor can have and how realistic initial conditions may be crucial to a successful and predictive theory of CCSNe. Finally, I conclude in §9 with a brief summary and closing remarks.

2. Historical perspective

The physical mechanism that drives CCSN explosions is a mystery that has endured for over 80 years. Fritz Zwicky and Walter Baade first identified and labelled supernovae in 1934 by measuring the distances to optical transients and determining that a subset of these were at much greater distance than common novae [20]. These so-called supernovae were incredibly more luminous and energetic than common novae, prompting Zwicky and Baade to postulate that the only physical process that could release so much energy is the gravitational collapse of the stellar core to a ball of neutrons at nuclear density, a neutron star. This prescient prediction is incredible
for two reasons: First, it is correct, or so we still think; and second, the neutron itself had only been discovered 2 years earlier [21]! While it is almost certainly the case that the energy reservoir that the CCSN mechanism draws upon is the gravitational binding energy liberated by the collapse of the stellar core to a neutron star, how that energy is used to power these explosions is still uncertain.

Perhaps the earliest plausible mechanism for translating the energy of collapse into explosion was the explosive thermonuclear burning of unspent fuel surrounding the collapsing core. This was the generally accepted paradigm around the time of Burbidge et al. [22], and expanded upon by Hoyle & Fowler [23]. Some of the first simulations of stellar core collapse in the early 1960s, however, revealed that nuclear burning during collapse could not liberate enough energy to drive an explosion. Further, the substantially sub-hydrostatic pressure of the collapsing inner core below the thermonuclear fuel actually resulted in an implosion as the explosive burning moved inwards chasing the collapsing core [24]. Instead, Colgate & White [24] proposed a different mechanism: the neutrinos diffusing out of the newborn neutron star could promptly energize the shock created by the stiffening of the inner core. Based on more detailed neutrino diffusion calculations, Arnett [25] cautioned that the considerable uncertainties in the EOS of hot dense matter and in the opacity of neutrinos under these conditions imply that neutrino heating may fail in some or all cases.

This was then largely the state of affairs for simulations of the CCSN mechanism for decades thereafter: state-of-the-art 1D simulations typically failed to explode via neutrino heating or were otherwise marginal at best. This allowed space for exploration of other more exotic mechanisms, the most successful of which was the magnetorotational mechanism pioneered by LeBlanc & Wilson [26] (see also e.g. [27]). While the rotational energy of a rapidly rotating protoneutron star (PNS) can far exceed that required to drive a CCSN explosion and can be readily transferred into powering an explosion, the magnetorotational mechanism is very likely not the prosaic CCSN mechanism, as it is now thought that the vast majority of massive stellar cores do not rotate nearly rapidly enough. The neutrino heating mechanism was revived, however, in the mid-1980s by Bethe & Wilson [28] and Bruenn [29] as the ‘delayed’ neutrino heating mechanism. Basically, it was realized that the explosion does not need to occur promptly following core bounce, but the stalled shock could be reinvigorated by the neutrino heating some hundreds of milliseconds later. As the legend goes, it was Jim Wilson who made this discovery by accidentally forgetting to stop one of his simulations, allowing it to run much longer than he had intended; the stalled shock was eventually revived and an explosion ensued.

The delayed neutrino heating mechanism has been the main focus of research into the CCSN mechanism since the advances of the mid-1980s. The success of the mechanism, however, has largely been marginal. As the fidelity of the physics included in calculations improved, 1D simulations increasingly failed to explode (e.g. [30]). In the mid-1990s, advancing computer power allowed the first realistic 2D simulations of the CCSN mechanism. What resulted was the realization of the remarkable impact of non-spherical instabilities, in particular convection. In the 2D simulations, convection occurred in two regions: first, steep lepton number gradients in the PNS drove convection; and, second, entropy gradients established by neutrino heating drove convection in the gain region immediately behind the stalled shock. The convection in the PNS is beneficial to shock revival and explosion because it enhances the emergent neutrino flux, increasing the heating in the gain region (e.g. [31]), although more recent studies have shown that this effect is less substantial in 2D [32]. Gain region convection increases the average amount of time it takes the accretion flow to transit the gain region, increasing the neutrino heating efficiency slightly [33], and also drives turbulence that aids shock expansion [34]. Additionally, the standing accretion shock instability (SASI) discovered by Blondin et al. [35] can also aid explosions in 2D (and possibly 3D) by leading to a secular growth of the typical shock radius (e.g. [36]).

The last several years have seen the emergence of high-fidelity 3D CCSN simulations from multiple groups (e.g. [37–43]). The results from these initial 3D simulations are encouraging and enticing, but also generally quite different from comparable 2D simulations in fundamentally important ways. The inescapable conclusion is that, unsurprisingly, the CCSN mechanism must
be studied in 3D. What we are learning from recent 3D simulations of CCSNe will be the subject of more thorough discussion in §6.

3. Modern view of stellar core collapse

Massive stars live fast and die young. Here, I briefly review the salient points of massive stellar evolution as they relate to the CCSN mechanism. For a thorough and current review of massive stellar evolution see Langer [44]. Following main-sequence lifetimes of only millions of years, stars more massive than about 8–10 $M_\odot$ go through multiple epochs of core and shell burning of ever heavier elements ultimately culminating in Si ‘burning’ to form cores of iron. The iron cores in massive stars are supported by electron degeneracy pressure, strongly cooled via neutrinos, and surrounded by burning shells of Si, O, C and so on. The complex, quasi-equilibrium Si shell burning continues to grow iron cores up to the effective Chandrasekhar mass, which is in general a function of electron fraction and core entropy [45]:

$$M_{\text{Ch}} \approx 1.44 \left( \frac{Y_e}{0.5} \right)^2 \left[ 1 + \left( \frac{s_e}{\pi Y_e} \right)^2 \right] M_\odot,$$

(3.1)

where $Y_e$ is the number of electrons per baryon and $s_e$ is the entropy of electrons in the core. This yields critical iron core masses ranging from about 1.3 $M_\odot$ to about 1.7 $M_\odot$. Once an iron core attains this critical mass, unstable gravitational collapse ensues. It is interesting to note that this is a true example of the gravitational instability originally envisaged by Chandrasekhar in deriving the critical mass of a self-gravitating star supported by electron degeneracy pressure [46]. Though usually associated with white dwarfs and Type Ia supernovae, the C/O white dwarf progenitors of thermonuclear supernovae never actually attain the critical maximum mass; C ignites near the star’s centre just prior to that.

The collapse of the critical-mass iron core accelerates rapidly. This runaway is driven principally by photodissociation of iron-peak nuclei, $\text{Fe}(\gamma, \alpha)$, and by electron captures:

$$p + e^- \rightarrow n + \nu_e.$$

(3.2)

The photodissociation reactions absorb about 8 MeV per nucleon, while the electron captures rob the core of electron degeneracy pressure support. Both processes speed the collapse of the core, driving core densities and temperatures higher and higher. The rates of both processes increase dramatically with density and temperature, setting up a true runaway collapse. The inner part of the core, about 0.4 to 0.6 $M_\odot$, is in sonic contact and collapses homologously, while the outer core collapses supersonically. The rapid infall proceeds until the central density exceeds that of nuclear matter, $\rho_{\text{nuc}} \sim 2 \times 10^{14}$ g cm$^{-3}$, while the ever increasing rate of electron capture on heavy nuclei drives $Y_e$ down to $\lesssim 0.3$. At such densities, and implied inter-nucleon spacings, the repulsive core of the strong nuclear force kicks in, effectively stiffening the EOS dramatically. The collapse of the inner core is halted suddenly, launching a strong shock wave into the still collapsing outer core in a sequence of events referred to as core ‘bounce.’ The shock propagates outwards rapidly at first through the outer core, but strong neutrino cooling behind the shock and photodissociation of iron-peak nuclei cause the shock to stall around a radius of 150 km. The photodissociation of iron requires about $1.5 \times 10^{51}$ erg per 0.1 $M_\odot$, robbing the shock of most of the energy liberated by the gravitational collapse of the inner core.

The inner supranuclear-density core has regained quasi-hydrostatic equilibrium and is made mostly of neutrons: a newborn PNS. This whole process of iron core collapse to bounce, beginning with a Chandrasekhar-mass core with radius of about 2000 km and ending with a PNS with a radius of about 50 km, takes a fraction of a second, generally around 0.25 s. The stalled bounce shock becomes, in essence, a standing accretion shock as its outward motion is halted and the remainder of the collapsing core, and stellar mantle above it, continue to fall down onto it. The supernova problem then is: What revives this stalled shock causing it to move outwards explosively and unbind the envelope of the star, driving a powerful stellar explosion with the observable
characteristics of known CCSNe? The initial conditions for the supernova problem are then as described above: a hot, strongly neutrino cooling PNS surrounded by a post-shock settling region bounded by a standing accretion shock with the collapsing star beyond it.

4. Neutrino-driven explosions in one dimension

The last couple of decades in research into the CCSN mechanism have focused on the neutrino mechanism. The gravitational collapse liberates an enormous amount of gravitational binding energy,

\[ \Delta E_b = GM_{\text{core}}^2 \left( \frac{1}{R_{\text{PNS}}} - \frac{1}{R_{\text{core}}} \right) \]

\[ \approx 1.2 \times 10^{53} \text{ erg} \left( \frac{M_{\text{core}}}{1.5 M_\odot} \right)^2 \left( \frac{50 \text{ km}}{R_{\text{PNS}}} - \frac{2000 \text{ km}}{R_{\text{core}}} \right). \]  

(4.1)

This is a hundred times the energy needed to drive a canonically energetic CCSN with \( E_{\text{exp}} \sim 10^{51} \text{erg} = 1 \text{ B} \). The vast majority of this energy, however, will be radiated in the form of neutrinos. As Colgate & White [24] proposed, if a small yet sufficient fraction of this radiated energy could be reabsorbed by the post-shock plasma, the shock could be reinvigorated, resulting in an explosion. The prompt neutrino-driven mechanism as originally envisaged by Colgate & White fails for all but the lowest-mass progenitors, often associated with the so-called electron capture SNe. But as Bethe & Wilson [28] showed, neutrino-driven explosions could result at relatively ‘late’ times, a few hundred milliseconds following bounce. While the success of the delayed neutrino heating mechanism was significantly dependent upon input physics, and 1D simulations with accurate microphysics still tended to fail (cf. [25,29]), research on the neutrino mechanism has been spurred on by periodic success and advancement in multidimensional simulation (e.g. [36,47–50]).

The prospect of successful self-consistent multidimensional explosions via the neutrino mechanism raises an important question: Does the neutrino mechanism for CCSNe lead to predictions for observable metrics that agree with actual data? Given the great expense and difficulty in reaching the long time scales needed to address this question in 2D and 3D, most of the attempts to answer this question directly and systematically have come in 1D. Early efforts at this focused on artificially driven explosion with either momentum input (‘pistons’; e.g. [51]) or heat input (‘thermal bombs’; e.g. [52]) and aimed at predictions for nucleosynthesis. This approach, while very useful for understanding cosmic chemical evolution, can provide little constraint on the CCSN mechanism itself. More recently, several studies have sought to systematically explore the observable properties of the neutrino mechanism (e.g. [53–59]). Using a simple neutrino leakage parametrization, O’Connor & Ott [53] explored the dependence of black hole formation time on progenitor structure. They introduced the parameter of the progenitor core compactness,

\[ \xi_M = \frac{M/M_\odot}{R(M_{\text{enclosed}} = M)/1000 \text{ km}} \bigg|_{t=t_{\text{bounce}}}, \]  

(4.2)

as a key parameter in understanding the outcome of core collapse. O’Connor & Ott showed that the core compactness correlated strongly with the time until black hole formation in failed explosions: more compact cores collapse to black holes sooner. They also showed that the core compactness is correlated with ‘explodability’: less compact cores require relatively less enhancement of the neutrino heating rate to achieve explosions. Using a new two-moment neutrino transport method, O’Connor & Ott [55] extended this work to show that the neutrino emission from failed CCSNe is also tightly correlated with compactness. Compactness alone, however, does not tell the whole story of explodability [57], particularly in 2D [49,50,60].

Ugliano et al. [54] performed a more detailed study of the neutrino mechanism across a wide range of progenitors using an approximate neutrino transport method. Ugliano et al. found that the neutrino mechanism, appropriately parametrized, can roughly reproduce key observable features of CCSN populations such as remnant mass distributions, including the mass gap
between neutron stars and black holes, and explosion energies approximately up to $2 \times 10^{51}$ erg. These authors also introduce the idea of ‘islands’ of black hole formation as a function of progenitor zero age main-sequence (ZAMS) mass. That is, the outcome of core collapse, be it explosion accompanied by a neutron star remnant, or failure and black hole formation, is non-monotonic with progenitor mass. Ugliano et al. [54] also show that the outcome is also not simply determined by progenitor core compactness, as suggested by O’Connor & Ott [53]. Ertl et al. [57] introduce a two-parameter predictor for the ‘explodability’ of a given progenitor model based on the progenitor mass coordinate at which the entropy, $s$, exceeds $4k_B$ per baryon, $M_4$, and the gradient of the enclosed mass at this location, $\mu_4 \equiv (dn/M_\odot)/(dr/1000\text{ km})|_{s=4}$. Predictions of explodability based upon these two parameters, Ertl et al. argue, more reliably reproduce the results of 1D explosions using approximate neutrino transport. In a sweeping investigation, Sukhbold et al. [58] demonstrate that parametrized 1D neutrino-driven explosions can yield populations of simulated CCSN that resemble in detail real observational data, including the neutron star mass distribution function and bolometric light curves. Figure 1 shows the results for neutron star and black hole formation from Sukhbold et al. [58] for several different parametrizations of the neutrino heating.

These works, and others in 1D, are encouraging for the neutrino mechanism for CCSNe. They imply that we are on the right track but they suffer in generality because 1D explosions must be artificially driven in some manner. And as Sukhbold et al. [58] show, the exact nature of the explosion parametrization can have a significant impact on the resultant predictions for CCSN populations. What is needed are self-consistent multidimensional CCSN simulations that span a similarly wide swath of the progenitor parameter space as the 1D studies (e.g. [61]).

5. Beyond spherical symmetry: self-consistent explosions in 2D

Even before the introduction of the first 2D simulations of CCSNe, it was realized that non-spherical instabilities and convection could be critically important to the explosion mechanism [62]. Early work focused on the role of convection in the PNS and the attendant enhancement of emergent neutrino luminosities [31,63,64], but early 2D simulations also showed the important role of neutrino-driven convection in the gain region just behind the stalled bounce shock [65]. Work in 2D developed rapidly while a persistent tension emerged: results from different groups...
Figure 2. Average shock radii from the 1D and 2D CCSN simulations of O’Connor & Couch [49] for four progenitors from [68], along with the 15 $M_{\odot}$ model from Woosley & Weaver [51]. Models s15, s20 and s25 all explode within about one second post-bounce.

disagreed, often qualitatively on even the basic question of whether a given progenitor model led to a successful explosion or not (e.g. [36,47,66,67]).

Recent work in 2D has seen the inclusion of ever more accurate numerical methods and microphysics and the tension among results from different groups is relaxing. In [49], we present 2D neutrino radiation–hydrodynamic simulations of CCSNe in the 12, 15, 20 and 25 $M_{\odot}$ progenitor models of Woosley & Heger [68]. The average shock radii from these simulations are shown in figure 2. We use a new explicit two-moment multidimensional neutrino transport solver based on the ‘M1’ closure [69,70]. We find successful neutrino-driven explosions for all of these progenitors except for the 12 $M_{\odot}$ star. The diagnostic explosion energies are still increasing at the end of our simulations at about one second post-bounce, but are in the range of 0.1–0.3 B. Interestingly, the ordering of explosion times and explosion energies is opposite from that predicted by artificially driven 1D explosions (see §4). Summa et al. [60] present 2D CCSN simulations using the Prometheus-VERTEX code with the same progenitors used in O’Connor & Couch [49]. They find explosions for all four progenitors, though the 12 $M_{\odot}$ star explodes very late. The ordering of explosion times for the 15, 20 and 25 $M_{\odot}$ progenitors is nearly the same between Summa et al. and O’Connor & Couch: higher-mass stars tend to explode earlier. The diagnostic explosion energies are also extremely similar.

Using the Castro adaptive mesh refinement (AMR) code with flux-limited diffusion (FLD) neutrino transport [71–73], the Princeton–LANL collaboration did not find that any of the same four [68] progenitors exploded in 2D [74]. A possible explanation for this qualitative difference might be due to the FLD approach used or, more probably, to the purely Newtonian gravity approximation made in [74]. In [49], we show in a controlled manner the positive impact that even approximate general relativistic (GR) gravity [75] can have on the explosion mechanism. This GR ‘effective potential’ approach is commonly used by other groups simulating CCSNe (e.g. [36,76,77]) and compares very well to fully GR simulations in 1D [78] and 2D [48]. Indeed, more recent results from the Princeton–LANL collaboration using their new code Fornax [50], which incorporates the GR effective potential, show explosions for the same progenitors as in [43]. Burrows et al. [50] even find that the 12 $M_{\odot}$ model of Woosley & Heger [68] fails to explode, as in [49].

The Chimera collaboration also finds successful 2D explosions in their most recent work [77,79]. They find robust explosions for all four of the 12, 15, 20 and 25 $M_{\odot}$ progenitors of Woosley & Heger [68]. Distinct from Summa et al. [60] and O’Connor & Couch [49], the explosions
all occur at nearly exactly the same time post-bounce and there is no period of shock recession as is seen in other 2D explosions [49,50,60]. The diagnostic explosion energies are also substantially higher in [77,79] than in other works, but this could be due to the earlier onset of explosion or, possibly, to the more detailed handling of the composition at low density by the Chimera collaboration.

Recent 2D CCSN simulation results are encouraging. Explosions are being found for progenitors across the expected range of initial masses and, more importantly, results from different groups using very different methods are beginning to agree not just qualitatively but quantitatively. Still, there remain important open questions. Can CCSN simulations reproduce the typical observed explosion energies? Only two [77,79] so far come close. Can CCSN simulations accurately predict the neutron star mass distribution? Current simulations universally show PNSs more massive than the canonical 1.4 $M_{\odot}$ for observed neutron stars. And, crucially, will the successes in 2D be recovered in fully 3D simulations of CCSNe? It is well known that 2D simulations are beset by drawbacks resulting from the artificial imposition of axisymmetry. The explosions are typically prolate, elongated along the symmetry axis, and may suffer from incorrect behaviour in 2D of the convection and turbulence (e.g. [80,81]).

### 6. Life and stellar death in the third dimension

Aided by Moore’s law, high-fidelity 3D CCSN simulations are now becoming computationally feasible. These simulations remain incredibly expensive on current petascale supercomputers, typically requiring tens of millions of core-hours per simulation. The lion’s share of the expense is due to neutrino transport. Formally, solving for the distribution functions of the neutrino radiation fields in 3D requires solving a 7D Boltzmann equation (e.g. [82]). This is still intractable for time-dependent simulations and so approximations must be made. One approach, which is technically exact but subject to statistical noise, is Monte Carlo transport [83,84], though this is still too expensive for sufficient particle counts to be feasible for 3D dynamic simulations. Far more common is some sort of moment-based approach to the Boltzmann equation that approximates the angular dependence of the radiation field as a truncated expansion with a suitable closure relation for higher-order moments. Truncation of this expansion following the zeroth moment (the energy density) yields a diffusion model (e.g. [77]). Truncation after the first moment yields a system of equations for the energy density and the radiation flux densities. Closure models for this approach include variable Eddington tensor (VET, [76,85]) or analytic ‘M1’ [70,86,87]. The latter method is particularly promising for 3D CCSN simulations because its explicit hyperbolic nature allows for better parallel scaling than implicit methods such as VET.

A great deal of discussion in the literature is devoted to the question of which approximation is more appropriate. It suffices to say that all CCSN simulation approaches currently used in 3D are approximate to a greater or lesser extent. And the approximations extend beyond just the method used for solving the neutrino transport to finite resolution, lack of MHD, assumption of nuclear statistical equilibrium (NSE) at low density or a simple ‘flashing’ approach [76] for the composition, etc. In short, all (3D) CCSN simulations employ approximations and there is no such thing as a ‘first-principles’ calculation. Instead, what should be sought is an accurate, self-consistent method that is free of ad hoc parameters. The neutrino transport methods mentioned above satisfy these requirements and generally compare very well to exact solutions of the Boltzmann equation for neutrinos [78,84,88–91]. And reasonable, well-justified approximations have enabled fully 3D simulations of the CCSN mechanism that have led to an incredible acceleration in our understanding of massive stellar death.

Before high-fidelity simulations became available, many of the first 3D CCSN simulations in the literature typically employed parametrized approaches to the neutrino physics, such as ‘lightbulb’ heating and cooling [92] or leakage schemes [93,94]. One of the key questions these parametrized simulations sought to address was: Is 3D the key to the CCSN mechanism? In other words, when compared with 2D, would 3D simulations explode more readily and yield explosions that better matched observational data? Early attempts to answer this question were
contradictory. Using a lightbulb approach, Nordhaus et al. [95] found that, all else being equal, the critical luminosity to achieve an explosion [96] was significantly lower in 3D than in 2D. In an attempt to reproduce these results, Hanke et al. [80] found little difference in the critical luminosity between 2D and 3D, though they point out significant dependence on the resolution of the simulation and some of their results even implied that 3D might be harder to explode. Burrows et al. [97] report that the gravity solver used in [95] had a significant inaccuracy that had since been corrected and the lightbulb study of Dolence et al. [98] found essentially no difference in the critical luminosity between 2D and 3D. Using relatively high resolution, I found in [81] that 3D simulations consistently and systematically required greater neutrino luminosity to explode than comparable 2D simulations. As also pointed out by Hanke et al. [80], reasons for this include the incorrect behaviour of turbulence in 2D and the tendency of the forced axisymmetry in 2D to exaggerate the amplitudes of the SASI. In a follow-up study using a more accurate leakage approach, we found in [99] that, again, 2D was artificially prone to explosion compared with 3D.

The first 3D simulations using high-fidelity, self-consistent neutrino transport in the literature confirmed this result. Using the Prometheus-VERTEX code, Tamborra et al. [100] and Hanke et al. [37] find no explosions in 3D for progenitor models that successfully explode in 2D. Melson et al. [40] report a successful 3D explosion for a low-mass iron core progenitor that also explodes in 1D. They find that the more physical behaviour of turbulence in 3D can actually aid energetic explosions for cases that actually achieve shock revival. While 3D explosions remained somewhat elusive for higher-mass progenitors, the study of Melson et al. [39] implies that many of the 3D simulations that fail are probably marginally close to reaching shock revival. They showed that including corrections to the neutrino cross sections due to nucleon strangeness [101] can lead to an explosion in 3D for a $20M_\odot$ progenitor. Janka et al. [5] also show that including moderately rapid rotation in a high-mass progenitor can trigger a neutrino-driven explosion. Without resorting to changes in the underlying physics such as the neutrino cross sections or progenitor rotation, Lentz et al. [41] find a successful explosion for a $15M_\odot$ progenitor, though the explosion occurs later than in their comparable 2D simulation and has a smaller explosion energy. Additionally, using a multidimensional M1 transport in fully GR 3D hydrodynamics, Roberts et al. [42] find 3D explosions for a $27M_\odot$ progenitor. These authors, however, report significant qualitative dependence on the grid resolution and geometry. Their high-resolution octant geometry case fails to explode entirely.

Clearly, the picture of the CCSN mechanism in 3D is not complete. Differences between different groups must be understood and the dependence on resolution, geometry and input
physics must be explored further in 3D in order to ascertain what the requirements for reliable 3D simulation are. In work in preparation [43], we have yet to find shock revival in 3D within 500 ms post-bounce for a 20\(M_\odot\) progenitor using high resolution and M1 neutrino transport. An image from one of the 3D simulations from this work is shown in figure 3. These simulations are ongoing and late-time shock revival is not precluded, as the comparable 2D simulation explodes at around 700 ms post-bounce.

7. Understanding success and failure: the turbulent frontier

To understand the difference in the likelihood for explosion between 2D and 3D, it is instructive to consider why 2D (and 3D) explode at all where 1D almost always fails. The standard picture for why multidimensionality aids CCSN explosion is as follows. First, convection in the PNS, driven in large part to a negative lepton gradient, enhances the emergent neutrino luminosity by dredging up trapped neutrinos and hardening the emitted neutrino spectra. PNS convection enhances heating in the gain region and, in extreme cases, can actually lead to explosions in 1D [62]. Net neutrino heating drives convection in the gain region that, on average, increases the typical matter dwell times there [98], thus increasing the heating rate. Finally, the SASI can drive shock expansion independent of neutrino heating [35,102,103]. All of these effects act to enhance the efficiency of neutrino heating: a larger fraction of the radiated neutrino energy is absorbed by matter in the gain region. This standard picture, then, implies that the total amount of neutrino energy that must be absorbed in order to achieve an explosion is roughly the same between 1D and 2D/3D; it is just that 2D/3D are more efficient at reaching that threshold energy.

Despite this being the accepted wisdom in the field, the standard picture of why 2D/3D succeed where 1D does not had not been put to a controlled test until [34]. Here, using a neutrino leakage approach in which we could tune the heating efficiencies, we found that 3D explosions were achieved at half the amount of integrated neutrino energy absorption as comparable 1D simulations. This implied that the standard picture was missing something: it was not simply that multidimensional effects lead to more efficient neutrino heating. The threshold for explosion, in terms of total heating, was lower in 3D (and 2D) than in 1D. In [34], we argued that the missing link was the action of strong turbulence in the gain region providing another form of pressure support to aid shock expansion. This work extended to 3D numerous earlier studies in 2D showing the importance of non-radial motion in the gain region to shock expansion [47,65,104,105].

The importance of turbulence in the CCSN mechanism was examined rigorously by Murphy & Meakin [106] and Murphy et al. [107]. In particular, Murphy et al. [107] point out that the effective turbulent pressure aids the thermal pressure in pushing the shock out. Murphy et al. [107] show that, in an angle-averaged sense, the shock jump conditions in 3D CCSNe are approximately

\[
P_d + \rho_d v_d^2 + \rho_d \rho_{\text{turb}} = \rho_u v_u^2, \tag{7.1}
\]

where \(\rho_d/\rho_u\) is the density just downstream/upstream of the shock and \(v_d/\rho_u\) is the radial velocity just downstream/upstream of the shock. \(R_{\text{rr}}\) is the radial–radial component of the Reynolds stress tensor, defined as \(R_{ij} = \gamma_{ij} \beta_{ij}\) where \(\gamma_{ij}\) are the velocity perturbations to the background mean flow. The third term on the LHS of equation (7.1) characterizes the turbulent pressure and is absent from the purely 1D shock jump conditions. Murphy et al. [107], and Couch & Ott [34] in a much larger set of 3D CCSN simulations, find that including this additional turbulent term is critical to predicting the average shock radius based on angle-averaged quantities. Figure 4 (from [34]) shows that neglecting the turbulent pressure in the shock jump conditions leads to a dramatic underestimate of the average shock radius (figure 4c). Figure 4 also shows the magnitude of the radial–radial Reynolds stress as well as the turbulent pressure, scaled to the background thermal pressure. The latter can be as large as 50% of the thermal pressure, implying that the turbulent pressure plays a leading-order role in the dynamics of the CCSN mechanism.

The turbulence is extremely efficient at aiding shock expansion. Expressing the turbulent pressure via a gamma-law EOS, \(P_{\text{turb}} = (\gamma_{\text{turb}} - 1)\rho e_{\text{turb}},\) where \(e\) is the specific energy of turbulent motion, the anisotropy of the turbulence in the CCSN gain region implies \(\gamma_{\text{turb}} \approx 2\), much larger
Figure 4. Quantities demonstrating the strength and importance of turbulence in 3D CCSNe: (a) radial–radial component of the Reynolds stress; (b) turbulent pressure scaled to the background thermal pressure; (c) excess in the shock jump conditions (cf. equation (7.1)) scaled to the upstream ramp pressure. In this figure, $\rho_0$ and $P_0$ are the spherically averaged density and thermal pressure, and $r_{sh}$ is the average shock radius. From Couch et al. [108].

than the thermal pressure $\gamma_{\text{therm}} \gtrsim \frac{4}{3}$ [109]. This means that per unit specific energy the turbulence yields a greater pressure to aid shock expansion.

Considering the important role of turbulence in establishing the dynamics of CCSN shock also provides a powerful means to understand the difference between the results of 2D and 3D turbulence. For 3D hydrodynamical turbulence, turbulent kinetic energy ‘cascades’ from some large driving scale down to smaller spatial scales until ultimately reaching the dissipation scale where the turbulent motions are converted to heat [110]. Turbulence in 2D simulations, however, exhibits an inverse turbulent cascade wherein turbulent kinetic energy moves from the driving scale to larger scales. In [34], we show that turbulent energy on large scales is correlated with likelihood for explosion, which is a reflection that explosions in 3D are typically triggered by the appearance of the first critically buoyant neutrino-heated plume (e.g. [81,98,111,112]). Thus in 2D, the inverse turbulent cascade tends to make conditions more favourable for the appearance of critically buoyant plumes that can trigger unstable shock expansion. This is an artefact of the forced symmetry in 2D and makes 2D CCSN simulations artificially prone to explosion.

The importance of turbulence in the CCSN mechanism also presents a significant computational challenge. The strength of turbulence is often described by the dimensionless Reynolds number, the ratio of inertial to viscous forces. The neutrino-heated gain region in CCSNe is highly turbulent. In the inviscid limit, the physical Reynolds numbers could be as large as
Numerous recent works have pointed out that the larger numerical viscosities due to finite resolution in current 3D CCSN simulations is such that the effective numerical Reynolds numbers may only be a few hundred [34,109,113,114], arguably not even turbulent. This could have a significant impact on the dynamics of the simulations, including a ‘bottleneck’ effect preventing an efficient cascade of turbulent kinetic energy from large to small scales [80,81,113,114]. As the transition from stalled shock to explosion in 3D is attended by the appearance of the large-scale buoyant plumes behind the shock [98,112], this bottleneck could have a crucial impact on the qualitative outcome of CCSN simulations. Radice et al. [109,115] find that capturing the inertial range of the turbulent cascade in 3D simulations of the CCSN mechanism may require an order-of-magnitude better resolution than current high-fidelity simulations. Such high resolution, however, may not be necessary. The drag exerted by neutrinos in the semi-transparent gain region acts as an effective viscosity reducing the Reynolds number, while magnetic fields will exert a tension that results in further reduction. While the numerical Reynolds number achieved in a given simulation is difficult to determine, it could be that direct numerical simulation of CCSN turbulence is feasible. More investigation of CCSN turbulence in high-fidelity 3D simulations is needed to settle these issues.

Turbulence and the role of the ‘turbulent pressure’ as quantified by the post-shock Reynolds stress provide a useful lens through which to look at the CCSN mechanism, but this lens does not show the entire picture. Certain cases, particularly models in 3D that are characterized as being ‘SASI-dominant’ [116,117], may be dominated more by the action of the SASI and of single or few large-scale, buoyant neutrino-heated plumes. And even in ‘convection-dominated’ models, for which a turbulent description of the gain region may be more applicable, effects such as turbulent dissipation [118] may be more important than the turbulent pressure.

8. Back to the beginning: the importance of accurate initial conditions

Massive stars explode in nature routinely and energetically. While the study of the CCSN mechanism in 3D high-fidelity simulations is still young, early indications are that the emerging picture of broad success of the neutrino mechanism in 2D simulations may not be recovered in 3D, at least not entirely. This raises the concern that we may still be missing some critical ingredient, or ingredients, to the CCSN mechanism. Fortunately, there are several possibilities. There remain significant uncertainties in crucial nuclear physics inputs such as the dense matter EOS and neutrino–matter interaction rates. As described above, we may not be treating the turbulence with sufficient accuracy in current 3D simulations, the implications of which are as yet not fully clear. Most high-fidelity 3D CCSN simulations neglect progenitor rotation and magnetic fields, yet all stars rotate and have magnetic fields. Also, there may be important aspects about the stellar progenitor models used in CCSN simulations that are uncertain or not handled with sufficient fidelity.

Stellar evolution is necessarily computed in 1D spherical symmetry. The enormous dynamic range of time scales makes fully multidimensional simulations of the entire evolution of a star intractable. The 1D stellar models, however, acknowledge that the breaking of spherical symmetry is a vital part of stellar evolution, in particular, for convective energy transport and various mixing processes (e.g. [68,119–122]). The convective speeds in the Si- and O-burning shells surrounding the pre-collapse iron core can reach hundreds of kilometres per second and are characterized by large spatial scale [123]. These convective fluctuations have traditionally been ignored in multidimensional CCSN simulations, in part because the stellar models are still fundamentally 1D, making it unclear how to handle the convective structure of the stars, and in part because it was thought they would not be important for the mechanism.

This latter assumption had never really been tested in a controlled manner. In [124], we studied whether pre-collapse velocity perturbations could impact the CCSN mechanism in 3D simulations using neutrino leakage. We added a simple non-radial velocity field to the Si/O layer surrounding the collapsing iron core in a 15\(M_\odot\) star and compared this model to a purely 1D
Figure 5. Radial velocity field from the 3D massive star simulation of Couch et al. [108] just prior to the onset of core collapse. This image shows the Si-burning-driven convection in the shell surrounding the iron core, around 1500 km in radius.

progenitor configuration in high-resolution 3D simulations. We found that the inclusion of large-scale velocity perturbations could trigger a successful explosion for a case that failed otherwise. The presence of the perturbations enhances the strength of turbulence in the gain region [34] and also increases the likelihood for the emergence of large-scale critically buoyant plumes that can drive shock expansions [111].

These results served as a reminder that the CCSN mechanism is essentially an initial value problem . . . and we may have problems with our initial values. Real massive stars are not spherically symmetric, particularly near core collapse, and this can have dramatic implications for the CCSN mechanism. This highlights the need for realistic 3D progenitor models for CCSN simulations. While the entirety of the evolution of a massive star cannot be attempted in 3D, short periods of minutes or hours can be. The final moments in the life of a massive star are extremely dynamic, at least in the core. The synthesis of the entire iron core lasts only a matter of days out of the millions of years of a massive star’s life. And the convective turnover times in the Si- and O-burning shells just prior to collapse are mere tens of seconds.

In [108], we present the first 3D simulation of the final minutes in the life of a massive star to the point of iron core collapse. An image from this simulation is shown in Figure 5. We had to make several approximations in order to achieve this. We reduced the 3D geometry to just a single octant, we used an approximate 21-isotope network instead of a full network capable of directly capturing the complex Si burning, and we had to parametrize the electron capture in the inner core in a manner that could have significant implications for the overall dynamics [125]. Despite these drawbacks, we find that, at the point of core collapse, the convection in the shells surrounding the iron core is strong, reaching speeds of hundreds of kilometres per second, and large in spatial scale. We used this progenitor in 3D CCSN mechanism simulations using neutrino leakage and found that the genuinely 3D structure had a positive effect on the strength of the explosion, though in this particular case the impact was not as dramatic as in [124].

Following our initial exploratory effort in [108], Müller et al. [125] studied the final minutes of O shell burning to the point of core collapse in an 18\( M_\odot \) star. They included the full star, rather than just an octant, and fixed the evolution of the inner core to follow the original 1D stellar evolution calculation, negating the need to resort to an ad hoc electron capture scheme. For this model, Müller et al. [125] find even stronger and larger-scale convective plumes that have a dominant quadrupole structure. In [112], the authors show that this realistic 3D structure can result in a qualitative impact on the CCSN mechanism. They find a successful explosion in this case where
the fully 1D model failed and, furthermore, they find that the strength of the CCSN explosion is tied directly to the strength of pre-collapse convection. This is an exciting result and makes it imperative that the 3D structure of massive stars at the point of core collapse be explored further. Such work could have a transformative impact on CCSN theory.

9. Summary

The theoretical study of the CCSN mechanism has entered an incredibly exciting stage. The advent of high-fidelity 3D simulations has already had an enormous impact on our understanding of massive stellar death and this is only the beginning. In this brief review, I have focused primarily on the neutrino mechanism for CCSNe. Recent work indicates that the dynamics in 3D are substantially different from comparable 2D simulations. This is due, in part, to the incorrect behaviour of turbulence in 2D (e.g. [34, 80, 107]). A significant development of the last few years is the importance of realistic 3D structure in the stellar progenitor models used in CCSN simulations. 3D stellar models suitable for use as the initial condition to CCSN mechanism simulations have already been shown to have a qualitative impact [34, 112, 124], and further work in this area could have a transformative impact on CCSN theory.

Several open questions remain. What level of physical fidelity is needed in the simulations to make accurate, robust predictions? A 3D CCSN simulation employing full Boltzmann neutrino transport and sufficient resolution to capture the inertial range of the turbulent cascade would require an exascale supercomputer. Such a supercomputer is still around 5 years away, but I would argue that significant progress towards a robust and predictive theory for CCSNe is possible with current petascale machines or the pre-exascale platforms that will be available soon. What are the sensitivities of CCSN observables to uncertainties in input nuclear physics? We are well aware of a strong dependence on the nature of the dense matter EOS but other sensitivities also exist, going all the way back to the nuclear reaction rates during stellar evolution. And what is the impact of rotation and magnetic fields on CCSNe? Some work on this issue has been done but much more is needed in high-fidelity 3D simulations.

Data accessibility. This article has no supporting data.

Competing interests. I declare I have no competing interests.

Funding. The author is supported in part by the US Department of Energy, Office of Science under award DE-SC0015904. Simulations described herein were completed with computer time provided by the Innovative and Novel Computational Impact on Theory and Experiment (INCITE) programme. This research used resources of the Argonne Leadership Computing Facility, which is a DOE Office of Science User Facility supported under Contract DE-AC02-06CH11357.


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